

# Package ‘Kendall’

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**Title** Kendall Rank Correlation and Mann-Kendall Trend Test

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**Depends** R (>= 2.1.0)

**Description** Computes the Kendall rank correlation and Mann-Kendall trend test. See documentation for use of block bootstrap when there is autocorrelation.

**Imports** boot

**LazyLoad** yes

**LazyData** yes

**Classification/ACM** G.3, G.4, I.5.1

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**License** GPL (>= 2)

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Kendall-package

*Kendall correlation and trend tests.*

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## Description

Compute Kendall's rank correlation and various trend tests.

## Details

Package: Kendall  
Type: Package  
Version: 2.2  
Date: 2011-05-16  
License: GPL (>= 2)  
LazyLoad: yes

If you just want to compute Kendall's tau or its significance level, the base function `cor` and `cor.test` are recommended. The purpose of this package is to implement the Mann-Kendall test, the seasonal Mann-Kendall trend test as well as computing the Kendall score. See documentation for Mann-Kendall on using the block bootstrap to deal with autocorrelation.

## Author(s)

A. I. McLeod

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## References

Hipel, K.W. and McLeod, A.I., (1994). Time Series Modelling of Water Resources and Environmental Systems.

## See Also

[cor](#), [cor.test](#), [print.Kendall](#), [summary.Kendall](#), [MannKendall](#), [SeasonalMannKendall](#)

## Examples

```
library(boot)
data(PrecipGL)
MKtau<-function(z) MannKendall(z)$tau
tsboot(PrecipGL, MKtau, R=500, l=5, sim="fixed")
```

---

GuelphP

*Phosphorous Concentrations in Speed River, Monthly*

---

### Description

Monthly time series of phosphorous (P) concentrations in mg/l, Speed River, Guelph, Ontario, 1972.1-1977.1."

### Usage

```
data(GuelphP)
```

### Format

The format is: Time-Series [1:72] from 1972 to 1978: 0.47 0.51 0.35 0.19 0.33 NA 0.365 0.65 0.825 1 ... - attr(\*, "title")= chr "Phosphorous Data,Speed River,Guelph,1972.1-1977.1"

### Details

See Hipel and McLeod (2005, pp.707-709 and p.986). Hipel and McLeod used intervention analysis to estimate the effect of a phosphorous removal scheme. Intervention analysis was also used to estimate the missing values.

### Source

Hipel, K.W. and McLeod, A.I., (2005). Time Series Modelling of Water Resources and Environmental Systems. Electronic reprint of our book originally published in 1994. <http://www.stats.uwo.ca/faculty/aim/1994Book/>.

### Examples

```
data(GuelphP)
plot(GuelphP)
#replace missing values in the series with estimates obtained from Hipel and McLeod
# (2005, p. 986)
missingEst<-c(0.1524, 0.2144, 0.3064, 0.1342)
GuelphP2<-GuelphP
GuelphP2[is.na(GuelphP)]<-missingEst
```

Kendall

*Kendall rank correlation***Description**

Computes the Kendall rank correlation and its p-value on a two-sided test of H0: x and y are independent. If there are no ties, the test is exact and in this case it should agree with the base function `cor(x,y,method="kendall")` and `cor.test(x,y,method="kendall")`. When there are ties, the normal approximation given in Kendall is used as discussed below. In the case of ties, both Kendall and `cor` produce the same result but `cor.test` produces a p-value which is not as accurate

**Usage**

```
Kendall(x, y)
```

**Arguments**

x	first variable, a vector
y	second variable, a vector the same length as x

**Details**

In many applications x and y may be ranks or even ordered categorical variables. In our function x and y should be numeric vectors or factors. Any observations corresponding to NA in either x or y are removed.

Kendall's rank correlation measures the strength of monotonic association between the vectors x and y. In the case of no ties in the x and y variables, Kendall's rank correlation coefficient, tau, may be expressed as  $\tau = S/D$  where

$$S = \sum_{i < j} (\text{sign}(x[j] - x[i]) * \text{sign}(y[j] - y[i]))$$

and  $D = n(n - 1)/2$ . S is called the score and D, the denominator, is the maximum possible value of S. When there are ties, the formula for D is more complicated (Kendall, 1974, Ch. 3) and this general formula for ties in both rankings is implemented in our function.

The p-value of tau under the null hypothesis of no association is computed by in the case of no ties using an exact algorithm given by Best and Gipps (1974).

When ties are present, a normal approximation with continuity correction is used by taking S as normally distributed with mean zero and variance  $\text{var}(S)$ , where  $\text{var}(S)$  is given by Kendall (1976, eqn 4.4, p.55). Unless ties are very extensive and/or the data is very short, this approximation is adequate. If extensive ties are present then the bootstrap provides an expedient solution (Davis and Hinkley, 1997). Alternatively an exact method based on exhaustive enumeration is also available (Valz and Thompson, 1994) but this is not implemented in this package.

An advantage of the Kendall rank correlation over the Spearman rank correlation is that the score function S is nearly normally distributed for small n and the distribution of S is easier to work with.

It may also be noted that usual Pearson correlation is fairly robust and it usually agrees well in terms of statistical significance with results obtained using Kendall's rank correlation.

An error is returned if  $\text{length}(x)$  is less than 3.

### Value

A list with class `Kendall` is returned with the following components:

<code>tau</code>	Kendall's tau statistic
<code>s1</code>	two-sided p-value
<code>S</code>	Kendall Score
<code>D</code>	denominator, $\tau=S/D$
<code>varS</code>	variance of S

### Note

Generic functions `print.Kendall` and `summary.Kendall` are provided.

If you want to use the output from `Kendall`, save the result as in `out<-Kendall(x,y)` and then select from the list `out` the value(s) needed.

### Author(s)

A.I. McLeod, [aim@uwo.ca](mailto:aim@uwo.ca)

### References

Best, D.J. and Gipps, P.G. (1974), Algorithm AS 71: The Upper Tail Probabilities of Kendall's Tau Applied Statistics, Vol. 23, No. 1. (1974), pp. 98-100.

Davison, A.C. and Hinkley, D.V. (1997) Bootstrap Methods and Their Application. Cambridge University Press.

Kendall, M.G. (1976). Rank Correlation Methods. 4th Ed. Griffin.

Hill, I.D. (1973), Algorithm AS 66: The Normal Integral Applied Statistics, Vol. 22, No. 3. (1973), pp. 424-427.

Valz, P. (1990). Developments in Rank Correlation Procedures with Applications to Trend Assessment in Water Resources Research, Ph.D. Thesis, Department of Statistical and Actuarial Sciences, The University of Western Ontario.

Valz, P.D. and Thompson, M.E. (1994), Exact inference for Kendall's S and Spearman's rho. Journal of Computational and Graphical Statistics, 3, 459-472.

### See Also

[cor](#), [print.Kendall](#), [summary.Kendall](#), [MannKendall](#), [SeasonalMannKendall](#)

**Examples**

```
#First Example
#From Kendall (1976, p.42-43, Example 3.4)
A<-c(2.5,2.5,2.5,2.5,5,6.5,6.5,10,10,10,10,10,14,14,14,16,17)
B<-c(1,1,1,1,2,1,1,2,1,1,1,1,1,1,1,2,2,2)
summary(Kendall(A,B))
#Kendall obtains S=34, D=sqrt(116*60), tau=0.41

#Second Example
#From Kendall (1976, p.55, Example 4.3)
x<-c(1.5,1.5,3,4,6,6,6,8,9.5,9.5,11,12)
y<-c(2.5,2.5,7,4.5,1,4.5,6,11.5,11.5,8.5,8.5,10)
summary(Kendall(x,y))
#Kendall obtains S=34 and Var(S)=203.30
```

---

MannKendall

*Mann-Kendall trend test*


---

**Description**

This is a test for monotonic trend in a time series  $z[t]$  based on the Kendall rank correlation of  $z[t]$  and  $t$ .

**Usage**

```
MannKendall(x)
```

**Arguments**

$x$  a vector of data, often a time series

**Details**

The test was suggested by Mann (1945) and has been extensively used with environmental time series (Hipel and McLeod, 2005). For autocorrelated time series, the block bootstrap may be used to obtain an improved significance test.

**Value**

A list with class Kendall.

tau	Kendall's tau statistic
s1	two-sided p-value
S	Kendall Score
D	denominator, $\tau=S/D$
varS	variance of S

Generic function print.Kendall and summary.Kendall are provided to print the output.

**Note**

If you want to use the output from MannKendall, save the result as in `res<-MannKendall(x,y)` and then select from the list `res` the value(s) needed.

**Author(s)**

A.I. McLeod, [aimcleod@uwo.ca](mailto:aimcleod@uwo.ca)

**References**

Davison, A.C. and Hinkley, D.V. (1997) *Bootstrap Methods and Their Application*. Cambridge University Press.

Hipel, K.W. and McLeod, A.I., (2005). *Time Series Modelling of Water Resources and Environmental Systems*. Electronic reprint of our book originally published in 1994. <http://www.stats.uwo.ca/faculty/aim/1994Book/>.

Mann, H.B. (1945), Nonparametric tests against trend, *Econometrica*, 13, 245-259.

**See Also**

[Kendall](#), [SeasonalMannKendall](#) [tsboot](#)

**Examples**

```
# Annual precipitation entire Great Lakes
# The time series plot with lowess smooth suggests an upward trend
# The autocorrelation in this data does not appear significant.
# The Mann-Kendall trend test confirms the upward trend.
data(PrecipGL)
plot(PrecipGL)
lines(lowess(time(PrecipGL),PrecipGL),lwd=3, col=2)
acf(PrecipGL)
MannKendall(PrecipGL)
#
#Use block bootstrap
library(boot)
data(PrecipGL)
MKtau<-function(z) MannKendall(z)$tau
tsboot(PrecipGL, MKtau, R=500, l=5, sim="fixed")
#
# Deseasonalize a monthly time series and use the block bootstrap
library(boot)
data(manus)
z<-matrix(manus, ncol=12, byrow=12)
zm<-apply(z, MARGIN=2, FUN=mean)
zs<-apply(z, MARGIN=2, FUN=sd)
z2<-sweep(z, MARGIN=2, STATS=zm) #subtract monthly means
z3<-sweep(z2, MARGIN=2, STATS=zs, FUN="/") #divide by monthly sd
zds<-c(t(z3))
attributes(zds)<-attributes(manus)
plot(zds)
```

```
#do Mann-Kendall trend test
MannKendall(zds)
#check robustness by applying block bootstrap
MKtau<-function(z) MannKendall(z)$tau
tsboot(zds, MKtau, R=500, l=12, sim="fixed")
#the significance level has dropped!
#it is significant now at about 2 sd
```

---

```
PrecipGL
```

```
Annual precipitation, inches, Great Lakes, 1900-1986
```

---

### **Description**

Annual precipitation, 1900-1986, Entire Great Lakes

### **Usage**

```
data(PrecipGL)
```

### **Format**

The format is: Time-Series [1:87] from 1900 to 1986: 31.7 29.8 31.7 33.1 31.3 ...

### **Source**

Appendix A of H. Roberts (1992) "Data Analysis for Managers" published by Scientific Press.

Original Source: Great Lake Water Levels, U.S. Dept of Commerce, Rockville MD U.S. Lake Survey, Detroit, MI, US Army Corps of Engineers

### **Examples**

```
data(PrecipGL)
plot(PrecipGL)
```

---

```
print.Kendall
```

```
print Method for Class 'Kendall'
```

---

### **Description**

The value of Kendall's tau and its two-sided p-value are displayed.

### **Usage**

```
## S3 method for class 'Kendall'
print(x, ...)
```



**Arguments**

x                    output from Kendall, MannKendall or SeasonalMannKendall  
...                  any additional arguments

**Value**

NULL

**Note**

If you want to use the output from Kendall or the other functions, save the result as in `res<-Kendall(x,y)` and then select from `res` the value(s) needed.

**Author(s)**

A.I. McLeod, aimcleod@uwo.ca

**See Also**

[summary.Kendall](#), [Kendall](#), [MannKendall](#), [SeasonalMannKendall](#), [summary.Kendall](#)

**Examples**

```
x<-1:10
y<-rnorm(10)
Kendall(x,y)
#is equivalent to
out<-Kendall(x,y)
print(out)
```

---

SeasonalMannKendall     *Mann-Kendall trend test for monthly environmental time series*

---

**Description**

Hirst et al. (1982) suggested this test for monthly water quality time series. The test is also discussed by Hipel and McLeod (2005).

The score is computed separately for each month.

The purpose of this test is to test for monotonic trend. A common misconception is to look for trends in the individual monthly time series. Usually this does not make a lot of sense, in the context of environmental time series, since if there is a real trend of interest in the series it would not be expected to be greatly changed by seasonality. If indeed one were interested in detecting a trend a particular month then one could use the MannKendall trend test for that particular month or group of months.

**Usage**

```
SeasonalMannKendall(x)
```

**Arguments**

x a vector or a time series comprised of consecutive monthly values

**Value**

A list with class Kendall.

tau	Kendall's tau statistic
s1	two-sided p-value
S	Kendall Score
D	Denominator, tau=S/D
varS	variance of S

Generic function print.Kendall and summary.Kendall are provided.

**Note**

If you want to use the output from SeasonalMannKendall, save the result as in `res<-SeasonalMannKendall(x,y)` and then select from the list `res` the value(s) needed.

**Author(s)**

A.I. McLeod, aimcleod@uwo.ca

**References**

Hirsch, R.M., Slack, J.R. and Smith, R.A. (1982), Techniques for trend assessment for monthly water quality data, Water Resources Research 18, 107-121.

Hipel, K.W. and McLeod, A.I., (2005). Time Series Modelling of Water Resources and Environmental Systems. Electronic reprint of our book originally published in 1994. <http://www.stats.uwo.ca/faculty/aim/1994Book/>.

**See Also**

[MannKendall](#)

**Examples**

```
#test for monotonic trend in monthly average river height data
#for the Rio Negro at Manaus. This data is included in the
#package boot.
library(boot)
data(manau)
SeasonalMannKendall(manau)
```

---

summary.Kendall      *summary Method for Class 'Kendall'*

---

### Description

In addition to the value of Kendall's tau and its two-sided p-value are displayed, the Kendall score, its variance and the value of the denominator for Kendall tau are printed.

### Usage

```
## S3 method for class 'Kendall'  
summary(object, ...)
```

### Arguments

object	output from Kendall, MannKendall or SeasonalMannKendall
...	any additional arguments

### Value

NULL

### Note

If you want to use the output from Kendall or the other functions, save the result as in `res<-Kendall(x,y)` and then select from `res` the value(s) needed.

### Author(s)

A.I. McLeod, aimcleod@uwo.ca

### See Also

[summary.Kendall](#), [Kendall](#), [MannKendall](#), [SeasonalMannKendall](#), [print.Kendall](#)

### Examples

```
x<-1:10  
y<-rnorm(10)  
out<-Kendall(x,y)  
out  
summary(out)
```

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