Package ‘DiscreteLaplace’

January 20, 2025

Type Package

Title Discrete Laplace Distributions

Version 1.1.1

Date 2016-04-29

Author Alessandro Barbiero <alessandro.barbiero@unimi.it>, Riccardo Inchingolo <dott.inchingolo_r@libero.it>

Maintainer Alessandro Barbiero <alessandro.barbiero@unimi.it>

Description Probability mass function, distribution function, quantile function, random generation and estimation for the skew discrete Laplace distributions.

License GPL

LazyLoad yes

Repository CRAN

Date/Publication 2016-05-01 00:46:21

NeedsCompilation no

Contents

DiscreteLaplace-package *Discrete Laplace Distributions*

Description

Probability mass function, distribution function, quantile function, random generation and sample estimation for two discrete skew Laplace distributions on integers. The skew discrete Laplace distributions here considered are that proposed by Kozubowski and Inusah (2006), henceforth referred to as DSL, and the alternative one proposed by Barbiero (2014), henceforth ADSL.

Details

Package:	DiscreteLaplace
Type:	Package
Version:	1.1.1
Date:	2016-04-29
License:	GPL
LazyLoad:	yes

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

Maintainer: Alessandro Barbiero <alessandro.barbiero@unimi.it>

References

T. J. Kozubowski, S. Inusah (2006) A skew Laplace distribution on integers, *Annals of the Institute of Statistical Mathematics*, 58: 555-571, http://dx.doi.org/10.1007/s10463-005-0029-1

A. Barbiero (2014) An alternative discrete Laplace distribution, *Statistical Methodology*, 16: 47-67, http://dx.doi.org/10.1016/j.stamet.2013.07.002

ddlaplace *Probability mass function, cumulative distribution function, quantile function and random generation of the DSL*

Description

The function computes the probability mass function, the cumulative distribution function, the quantile function of the DSL and implements random generation.

Usage

```
ddlaplace(x, p, q)
pdlaplace(x, p, q)
qdlaplace(prob, p, q)
rdlaplace(n, p, q)
```

Arguments

x	vector of quantiles
p	the first parameter p in $(0, 1)$ of the SDL
q	the second parameter q in $(0, 1)$ of the SDL
prob	vector of probabilities
n	number of observations

Details

The pmf of the SDL is given by

$P(X = x; p, q) = \frac{(1-p)(1-q)}{1-pq} p^x; x = 0, 1, 2, 3, \ldots$

$P(X = x; p, q) = \frac{(1-p)(1-q)}{1-pq} q^{|x|}; x = 0, -1, -2, -3, \ldots$

whereas the cumulative distribution function is given by

$F(x; p, q) = P(X \leq x) = \frac{(1-p)q^{-\lfloor x \rfloor}}{1-pq}, x < 0$

$F(x; p, q) = P(X \leq x) = 1 - \frac{(1-q)p^{\lfloor x \rfloor + 1}}{1-pq}, x \geq 0$

Value

`ddlaplace` returns the probability of `x`; `pdlaplace` returns the cumulate probability of `x`; `qdlaplace` returns the `prob`- quantile; `rdlaplace` returns a random sample of size `n` from DSL.

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

References

Tomasz J. Kozubowski, Seidu Inusah (2006) A skew Laplace distribution on integers, *Annals of the Institute of Statistical Mathematics*, 58: 555-571

See Also

`ddlaplace2`

Examples

```
# pmf
p<-0.7
q<-0.45
x<--10:10
prob<-ddlaplace(x, p, q)
plot(x, prob, type="h")
prob<-ddlaplace(x, q, p) # swap the parameters
plot(x, prob, type="h")
ddlaplace(-4:4, 1:9/10, 9:1/10) # letting p and q be vectors
# cdf
p<-0.2
q<-0.5
x<-c(-3, -1, pi)
pdlaplace(x, p, q)
# quantile function
p<-0.8
q<-0.4
prob<-c(0.2,0.5,0.8)
x<-qdlaplace(prob, p, q)
x # check
upper<-pdlaplace(x, p, q)
upper
lower<-pdlaplace(x-1, p, q)
lower
lower<=prob & prob<=upper
# random generation
n<-100
p<-0.3
q<-0.5
x<-rdlaplace(n, p, q)
x
t<-table(x)
t
plot(t)
```

---

ddlaplace2 *Probability mass function of the ADSL*

Description

The function computes the probability mass function, the cumulative distribution function, the quantile function of the ADSL and provides random generation of samples from the same model

Usage

```
ddlaplace2(x, p, q)
palaplace2(x, p, q)
pdlaplace2(x, p, q)
```

```
qdlaplace2(prob, p, q)
rdlaplace2(n, p, q)
```

Arguments

x	vector of quantiles
p	the first parameter p in $(0, 1)$ of the ADSL
q	the second parameter q in $(0, 1)$ of the ADSL
prob	vector of probabilities
n	number of observations

Details

The probability mass funtion of the ADSL distribution is given by:

$P(X = x; p, q) = \frac{\log p}{\log(pq)} q^{-(x+1)}(1 - q)$ for $x = \ldots, -2, -1$

and

$P(X = x; p, q) = \frac{\log q}{\log(pq)} p^{x}(1 - p)$ for $x = 0, 1, 2, \ldots$

Its cumulative distribution function is:

$F(x; p, q) = \frac{\log p}{\log(pq)} q^{-(\lfloor x \rfloor+1)}$ for $x < 0$

and

$F(x; p, q) = 1 - \frac{\log q}{\log(pq)} p^{(\lfloor x \rfloor+1)}$ for $x \geq 0$

Value

`ddlaplace2` returns the probability of `x`; `pdlaplace2` returns the cumulate probability of `x`; `qdlaplace2` returns the `prob`- quantile; `rdlaplace2` returns a random sample of size `n` from ADSL.

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

References

A. Barbiero, An alternative discrete Laplace distribution, *Statistical Methodology*, 16: 47-67

See Also

`ddlaplace`

Examples

```
# pmf
p <- 0.7
q <- 0.45
x <- -10:10
prob <- ddlaplace2(x, p, q)
plot(x, prob, type="h")
```

```
# swap the parameters
prob <- ddlaplace2(x, q, p)
plot(x, prob, type="h")
# letting p and q be vectors...
ddlaplace2(-4:4, 1:9/10, 9:1/10)
# cdf
pdlaplace2(x, p, q)
pdlaplace2(pi, p, q)
pdlaplace2(floor(pi), p, q)
# quantile function
qdlaplace(1:9/10, p, q)
# random generation
y <- rdlaplace2(n=1000, p, q)
plot(table(y))
```

`dlaplacelike2` *Log-likelihood function for the ADSL distribution*

Description

Log-likelihood function (changed in sign) for the ADSL distribution.

Usage

```
dlaplacelike2(par, x)
```

Arguments

par	the vector of parameters (p, q)
x	a vector of observations from ADSL

Value

The log-likelihood function with changed sign.

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

References

A. Barbiero, An alternative discrete Laplace distribution, *Statistical Methodology*, 16: 47-67

See Also

estdlaplace2, ddlaplace2

Examples

```
p <- 0.25
q <- 0.7
x <- rdlaplace2(n=100, p, q)
par <- estdlaplace2(x, "ML")
-dlaplacelike2(par, x) # greater than...
-dlaplacelike2(c(p, q), x)
```

Edlaplace *Moments of the discrete Laplace distribution*

Description

The function provides the expected value and the variance of the SDL, and the expectation of its absolute value.

Usage

```
Edlaplace(p, q)
```

Arguments

p	the first parameter, in $(0, 1)$, of the DSL
q	the second parameter, in $(0, 1)$, of the DSL

Details

$E(X; p, q) = \frac{1}{1-p} - \frac{1}{1-q} = \frac{p}{1-p} - \frac{q}{1-q}$,
$E(|X|; p, q) = \frac{q(1-p)^2+p(1-q)^2}{(1-qp)(1-q)(1-p)}$,
$V(X; p, q) = \frac{1}{(1-p)^2(1-q)^2}[\frac{q(1-p)^3(1+q)+p(1-q)^3(1+p)}{1-pq} - (p-q)^2]$

Value

A list of three items:

E1	expected value
E1a	expectation of the absolute value
V	variance

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

References

T. J. Kozubowski, S. Inusah (2006) A skew Laplace distribution on integers, *Annals of the Institute of Statistical Mathematics*, 58: 555-571

See Also

ddlaplace

Examples

```
# ex.1
p<-0.5
q<-0.4
Edlaplace(p, q)
# ex.2
p<-0.1
q<-0.9
Edlaplace(p, q)
```

Edlaplace2 *First- and second-order moments of ADSL*

Description

First- and second-order moment of the ADSL distribution.

Usage

```
Edlaplace2(p, q)
```

Arguments

p	the first parameter p, in $(0, 1)$, of the ADSL
q	the first parameter q, in $(0, 1)$, of the ADSL

Details

For the ADSL distribution,
$E(X; p, q) = \frac{\log q}{\log(pq)} \frac{p}{1-p} - \frac{\log p}{\log(pq)} \frac{1}{1-q}$
and
$E(X^2; p, q) = \frac{\log q}{\log(pq)} \frac{p(1+p)}{(1-p)^2} + \frac{\log p}{\log(pq)} \frac{1+q}{(1-q)^2}$

Value

A list containing the first- and the second-order moments of the ADSL distribution, E1 and E2.

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

References

A. Barbiero, An alternative discrete Laplace distribution, *Statistical Methodology*, 16: 47-67

See Also

estdlaplace2, loss, ddlaplace2

Examples

```
Edlaplace2(p=0.3, q=0.3)
Edlaplace2(p=0.3, q=0.6)
Edlaplace2(p=0.6, q=0.3)
Edlaplace2(p=0.6, q=0.6)
```

---

estdlaplace *Sample estimation for the DSL*

Description

The function provides the maximum likelihood estimates for the parameters of the DSL and the estimate of the inverse of the Fisher information matrix. The method of moments estimates of p and q coincide with the maximum likelihood estimates.

Usage

```
estdlaplace(x)
```

Arguments

x	a vector of observations from the DSL

Details

See the reference. If $\bar{x}^+ = \frac{1}{n}\sum_{i=1}^n x_i^+$, $\bar{x}^- = \frac{1}{n}\sum_{i=1}^n x_i^-$ where x^+ and x^- are the positive and the negative parts of x, respectively: $x^+ = x$ if $x \geq 0$ and zero otherwise, $x^- = (-x)^+$, then

$\hat{q} = \frac{2\bar{x}^-(1+\bar{x})}{1+2\bar{x}^- \bar{x}+\sqrt{1+4\bar{x}^- \bar{x}^+}}$, $\hat{p} = \frac{\hat{q}+\bar{x}(1-\hat{q})}{1+\bar{x}(1-\hat{q})}$

when $\bar{x} \geq 0$ and

$\hat{p} = \frac{2\bar{x}^+(1-\bar{x})}{1-2\bar{x}^+ \bar{x}+\sqrt{1+4\bar{x}^- \bar{x}^+}}$, $\hat{q} = \frac{\hat{p}-\bar{x}(1-\hat{p})}{1-\bar{x}(1-\hat{p})}$

when $\bar{x} \leq 0$.

Value

A list comprising

hatp	estimate of p
hatq	estimate of q
hatSigma	estimate of the inverse of the Fisher information matrix

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

References

T. J. Kozubowski, S. Inusah (2006) A skew Laplace distribution on integers, *Annals of the Institute of Statistical Mathematics*, 58: 555-571

See Also

ddlaplace

Examples

```
p<-0.6
q<-0.3
n<-20
x<-rdlaplace(n, p, q)
est<-estdlaplace(x)
est[1]
est[2]
est[3]
# increase n
n<-100
x<-rdlaplace(n, p, q)
est<-estdlaplace(x)
est[1]
est[2]
est[3]
# swap the parameters
x<-rdlaplace(n, q, p)
est<-estdlaplace(x)
est[1]
est[2]
est[3]
```

---

estdlaplace2 *Sample estimation for the ADSL*

Description

The function provides the point estimates for the parameters of the ASDL, resorting to four possible methods: method of moments, maximum likelihood method, method of proportion, modified method of moments. For details, please take a look at the references.

Usage

```
estdlaplace2(x, method = "M", err = 0.001, parml = c(exp(-1), exp(-1)))
```

Arguments

`x`	a vector of observations from the ADSL
`method`	`M` for the method of moments, `ML` for the maximum likelihood methods, `P` for the method of proportion, `MM` for the modified method of moments
`err`	a positive tolerance value, as small as possible, used in the definition of lower and upper bounds of the parameters p and q in the minimization algorithm utilized by the method of moments
`parml`	starting values for p and q in the optimization process for the maximum likelihood method

Value

a vector with the parameter estimates of p and q.

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

References

A. Barbiero, An alternative discrete Laplace distribution, *Statistical Methodology*, 16: 47-67

See Also

`dlaplacelike2`

Examples

```
p <- 0.4
q <- 0.6
x <- rdlaplace2(n=100, p, q)
est <- matrix(0, 5, 2)
est[1,] <- c(p,q)
est[2,] <- estdlaplace2(x, method="M")
est[3,] <- estdlaplace2(x, method="ML")
est[4,] <- estdlaplace2(x, method="P")
est[5,] <- estdlaplace2(x, method="MM")
dimnames(est)[[1]]<-c("true","M","ML","P","MM")
dimnames(est)[[2]]<-c("p","q")
xlim <- c(min(est[,1])*.98,max(est[,1])*1.02)
ylim <- c(min(est[,2])*.98,max(est[,2])*1.02)
plot(est, pch=19, col=1:5, xlim=xlim, ylim=ylim)
text(est, dimnames(est)[[1]], pos=3, col=1:5, cex= .75)
```

iFI *Inverse of Fisher Information matrix*

Description

Inverse of Fisher Information matrix for the DSL.

Usage

```
iFI(p, q)
```

Arguments

p	first parameter p, in $(0, 1)$, of the DSL
q	second parameter q, in $(0, 1)$, of the DSL

Value

The inverse of Fisher Information matrix. Take a look at the references for more details.

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

References

T. J. Kozubowski, S. Inusah (2006) A skew Laplace distribution on integers, *Annals of the Institute of Statistical Mathematics*, 58: 555-571

See Also

iFI2

Examples

```
p <- 0.2
q <- 0.8
iFI(p, q)
```

`iFI2` *Inverse of Fisher Information matrix*

Description

Inverse of Fisher Information matrix for the ADSL.

Usage

```
iFI2(p, q)
```

Arguments

p	first parameter p, in $(0, 1)$, of the ADSL
q	second parameter q, in $(0, 1)$, of the ADSL

Value

The inverse of Fisher Information matrix. Take a look at the references for more details.

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

References

A. Barbiero, An alternative discrete Laplace distribution, *Statistical Methodology*, 16: 47-67

See Also

ioFI2

Examples

```
p <- 0.2
q <- 0.8
iFI2(p, q)
```

ioFI2 *Inverse of the observed Fisher Information matrix*

Description

Inverse of the observed Fisher Information matrix computed on a random sample of ADSL values.

Usage

```
ioFI2(x)
```

Arguments

x	a vector of observations from the ADSL

Value

The inverse of the observed Fisher Information matrix.

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

References

A. Barbiero, An alternative discrete Laplace distribution, *Statistical Methodology*, 16: 47-67

See Also

iFI2

Examples

```
n <- 100
p <- 0.4
q <- 0.7
x <- rdlaplace2(n, p, q)
M <- ioFI2(x)
par <- estdlaplace2(x, "ML")
se <- diag(sqrt(M))
par # MLEs
se # standard errors
M # compare with  the inverse of Fisher Information matrix
iFI2(par[1], par[2])/n # with MLEs plugged in
iFI2(p, q)/n # or the true values
```

loss — *Loss function for the method of moments*

Description

A loss function used for the implementation of the method of moments (for the ADSL).

Usage

```
loss(par, x)
```

Arguments

`par`	the vector of parameters, `p` and `q`, of the ADSL
`x`	a vector of sample values from the ADSL

Value

The value $L = [E(X) - m_1(x)]^2 + [E(X^2) - m_2(x)]^2$, where m_1 and m_2 are the first- and second-order sample moments.

Author(s)

Alessandro Barbiero, Riccardo Inchingolo

References

A. Barbiero, An alternative discrete Laplace distribution, *Statistical Methodology*, 16: 47-67

See Also

`estdlaplace2`, `Edlaplace2`

Examples

```
p <- 0.3
q <- 0.7
x <- rdlaplace2(n=100, p, q)
par <- estdlaplace2(x, "M")
loss(par, x) # should be near zero
loss(c(p,q), x) # may be far greater than zero
```

Index